B.Sc. Part I

Paper I

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Theory of Relativity

Consequences of Lorentz Transformation equⁿs.

1.) Relativity of Simultaneity:-

The two events are said to be simultaneous if they occur at the same time. Let there be two frames of reference S and S', the latter moving with velocity v relative to former along +ve direction of x-axis. Assume that the two events occur simultaneously in frame S at Point P_1 and P_2.

Let (x_1, y_1, z_1, t_1) and (x_2, y_2, z_2, t_2) be the co-ordinates of measurements made by an observer at the origin of system S of the two events at P_1 and P_2 respectively. Since the events are simultaneous in frame S, therefore we have $t_1 = t_2$. If t_1' and t_2' are the corresponding times of the same two events with respect to the system S', then we have from Lorentz transformation equations.

$$t_1' = \frac{t_1 - vx_1/c^2}{\sqrt{(1 - v^2/c^2)}}$$

and

$$t_2' = \frac{t_2 - \frac{vx_2}{c^2}}{\sqrt{1 - v^2/c^2}}$$

$$\therefore t_2' - t_1' = \frac{t_2 - t_1}{\sqrt{(1 - v^2/c^2)}} - \frac{\frac{v}{c^2}(x_2 - x_1)}{\sqrt{(1 - v^2/c^2}}$$

$$= \frac{\frac{v(x_1 - x_2)}{c^2}}{\sqrt{(1 - \frac{v^2}{c^2})}} \quad \text{since } t_1 = t_2.$$

Thus if the events are simultaneous in frame S', t_1' must be equal to t_2' or $t_2' - t_1'$ must be equal to zero, but it is not so because x_1 is not equal to x_2. Therefore the same two events are not simultaneous in fram S'. As such "two events at different places P_1 and P_2 which are simultaneous for an observer at rest in frame S, are no longer simultaneous to an observer in frame S' having linear motion relative to S'.
This follows that ~~simultaneously~~ simultaneity is not absolute, but relative.

2.5 Einstein time dilation or Apparent retardation of clocks :-

Let there be two frames of references S and S'; S' moving with velocity v relative to S along (+ve) direction of x-axis, and suppose that this placed at a point x_1.

Let a clock be placed in reference system S which is at rest. Let this clock give a signal at time t_1 in system S and suppose that t_1' is the time measured by the observer is S' corresponding to the time t_1.
Then from Lorentz transformation equ^ns we have

$$t_1' = \frac{t_1 - \frac{v x_1}{c^2}}{\sqrt{(1-\beta^2)}} \quad \text{——— (i)}$$

If this clock again gives a signal at time t_2. Then corresponding time t_2' in frame S' is given by

$$t_2' = \frac{t_2 - \frac{vx_1}{c^2}}{\sqrt{(1-\beta^2)}} \quad \text{—— (2)}$$

Thus, the clock gives signal at an intervals $(t_2 - t_1) = \Delta t$ in system S, when this interval is measured from the moving system it is equal to $t_2' - t_1' = \Delta t'$. Therefore from equ^n (1) and (2) we have

$$(t_2' - t_1') = \frac{t_2 - t_1}{\sqrt{(1-\beta^2)}}$$

i.e.,

$$\Delta t' = \frac{\Delta t}{\sqrt{(1-\beta^2)}} = \Delta t\left(1 + \frac{v^2}{2c^2}\right) \quad \text{—— (3)}$$

Since $\beta = v/c$

i.e., $\Delta t' > \Delta t$

It is clear from expression (3). That time interval Δt appears to the moving observer to be dilated or lengthened by a factor

$$\frac{1}{\sqrt{(1-\beta^2)}} = \frac{1}{\sqrt{(1 = v^2/c^2}}$$

where v is the velocity of the moving observer.

Hence from above reasoning we may say "A moving clock always appears to go slow" consequently to the observer in motion the clock at rest appears to be retarded by the factor $\sqrt{(1-\beta^2)}$. This is apparent relation of clocks. The above reasoning may be summarised as a rule.

" Every clock appears to go at its fastest rate when it is at rest relative to the observer. Its rate appears to go on slowing by the factor

$\sqrt{(1-\beta^2)}\left\{=\sqrt{\left(1-\frac{v^2}{c^2}\right)}\right\}$ as its velocity v relative to the observer goes on increasing. "